Modeling and simulation of contact between rigid bodies

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Fluid model with moving rigid bodies

Navier-Stokes equations :

$$\left\{ \begin{array}{rcl} \rho_f \frac{du}{dt} + \rho_f (u \cdot \nabla) u - \nabla \cdot \sigma & = & 0_{\mathbb{R}^d}, \Omega, \\ \nabla \cdot u & = & 0, \quad \Omega, \end{array} \right.$$

where σ the total stress tensor :

$$\sigma = -p I_d + \mu [\nabla u + (\nabla u)^T].$$

Fluid definitions :

- $d = 2$ or $d = 3$, dimension.
- $\Omega \subseteq \mathbb{R}^d$, fluid domain.
- $u : \Omega \to \mathbb{R}^d$, velocity.
- $p : \Omega \to \mathbb{R}$, pressure.
- $\mu \in \mathbb{R}^+$, viscosity.
- $\rho_f \in \mathbb{R}^+$, density.
- $I_d \in \mathbb{M}_{d,d}$, identity tensor.

Fluid model with moving rigid bodies

The bodies can translate and rotate with gravity, fluid forces acting on them and contact, lubrication forces in body-body and body-wall interactions.

Newton equations :

$$m_i \frac{dU_i}{dt} = (m_i - \int_{B_i} \rho_f) g + F_i + F_i^c + F_i^l,$$

$$\frac{d[RJ_iR^T\omega_i]}{dt} = T_i + T_i^c + T_i^l,$$

i-**th body definitions :**
- $B_i \subseteq \mathbb{R}^d$, body domain.
- $m_i \in \mathbb{R}$, mass.
- $J_i \in \mathbb{R}^+$ if $d = 2$, else $J_i > 0 \in \mathbb{M}_{d,d}$, moment of inertia.
- $R \in \mathbb{M}_{d,d}$, rotation matrix.
- $U_i \in \mathbb{R}^d$, translational velocity.
- $\omega_i \in \mathbb{R}$ if $d = 2$, else $\omega_i \in \mathbb{R}^d$, angular velocity.
- $x_i^{CM} \in \mathbb{R}^d$, center of mass.

where $g \in \mathbb{R}^d$ is the gravity vector, $F_i^c, F_i^l \in \mathbb{R}^d$, T_i^c, $T_i^l \in \mathbb{R}^d$ the contact, lubrication forces and torque, and $F_i \in \mathbb{R}^d$, $T_i \in \mathbb{R}^d$ the hydrodynamic forces and torque acting on the body :

$$F_i = -\int_{\partial B_i} \sigma \cdot \overrightarrow{n} \quad \text{and} \quad T_i = -\int_{\partial B_i} (\sigma \cdot \overrightarrow{n}) \times (x_i - x_i^{CM}).$$

Fluid model with moving rigid bodies

Coupling condition on the velocities on ∂B_i :

$$u = U_i + \omega_i \times (x - x_i^{CM}).$$

Fluid-body problem :

$$\begin{cases} \rho_f \frac{du}{dt} + \rho_f (u \cdot \nabla) u - \nabla \cdot \sigma &=& 0_{\mathbb{R}^d}, \quad \text{in } \Omega, \\ \nabla \cdot u &=& 0, \quad \text{in } \Omega, \\ u &=& U_i + \omega_i \times (x - x_i^{CM}), \quad \text{on } \partial B_i, \\ m_i \frac{dU_i}{dt} &=& (m_i - \int_{B_i} \rho_f) g + F_i + F_i^c + F_i^l, \\ \frac{d[R J_i R^T \omega_i]}{dt} &=& T_i + T_i^c + T_i^l. \end{cases}$$

Decheng Wan, Stefan Turek. *Direct Numerical Simulation of Particulate Flow via Multigrid FEM Techniques and the Fictitious Boundary Method*, 2004.

Contact and lubrication forces

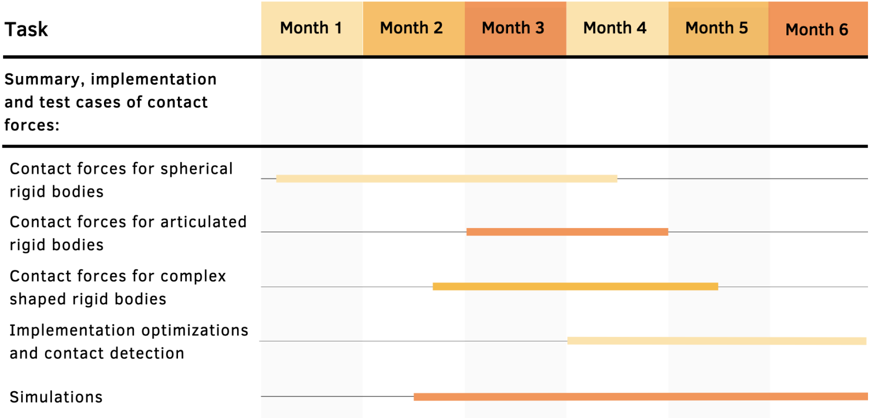
Contact forces

- Body surfaces are in direct contact.
- Approaches :
  - Contact model based on hard sphere approach. [4]
  - Hertzian contact theory. [6]
  - Contact model based on soft sphere approach. [5]

Lubrication forces

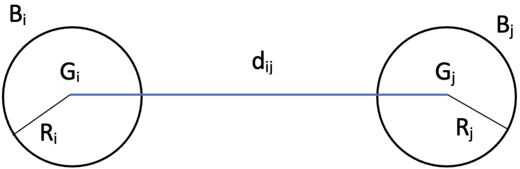
- Distance between body surfaces are very small.
- Ensure that hydrodynamic forces are resolved.
- Approaches :
  - Lubrication model with constant force. [4]
  - Lubrication model with repulsive force. [1]

Gantt chart



Full Gantt chart : https ://feelpp.github.io/swimmer/swimmer/latest/StageCeline/Introduction.html

Smooth collision : The velocities of the rigid bodies coincide at the points of contact.



We note $d_{ij} = ||G_i - G_j||_2$, distance between the disk mass centers.

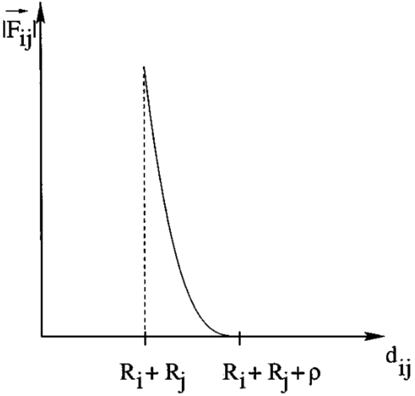
The repulsion force $\overrightarrow{F_{ij}}$ has to verify three properties :

1. $\overrightarrow{F_{ij}}$ is parallel to $\overrightarrow{G_i G_j}$.

R. Glowinski, T. W. Pan, T. I. Hesla, D. D. Joseph, and J. Périaux. *A Fictitious Domain Approach to the Direct Numerical Simulation of Incompressible Viscous Flow past Moving Rigid Bodies : Application to Particulate Flow*, 2000.

Collisions between spherical rigid bodies

2. $|\overrightarrow{F_{ij}}| = 0 \quad$ if $d_{ij} > R_i + R_j + \rho$, where ρ the range of the repulsion force.
3. For $R_i + R_j \leq d_{ij} \leq R_i + R_j + \rho$:



Repulsion force [1]

Collisions between spherical rigid bodies

Definition of $\overrightarrow{F_{ij}}$ for body-body interaction :

$$|\overrightarrow{F_{ij}}| = \begin{cases} 0\,, & \text{for} \quad d_{ij} > R_i + R_j + \rho, \\ \frac{1}{\epsilon}(G_i - G_j)(R_i + R_j + \rho - d_{ij})^2\,, & \text{for} \quad R_i + R_j \leq d_{ij} \leq R_i + R_j + \rho, \\ \frac{1}{\epsilon'}(G_i - G_j)(R_i + R_j - d_{ij})\,, & \text{for} \quad d_{ij} < R_i + R_j. \end{cases}.$$

where $(R_i + R_j + \rho - d_{ij})^2$ a quadratic activation term and $(G_i - G_j)$ gives the direction of the force.

The stiffness parameters are set to $\epsilon \approx h^2$ and $\epsilon^{'} \approx h$, if :

- $\rho \approx h$, h the mesh step.
- $\frac{\rho_b}{\rho_f} \approx 1$, ρ_b the body density and ρ_f the fluid density.
- The fluid is sufficiently viscous.

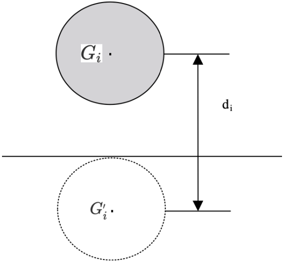
Decheng Wan, Stefan Turek. *Direct Numerical Simulation of Particulate Flow via Multigrid FEM Techniques and the Fictitious Boundary Method*, 2004.

Collisions between spherical rigid bodies

Definition of $\overrightarrow{F_i^W}$ for body-wall interaction :

$$|\overrightarrow{F_i^W}| = \begin{cases} 0\ , & \text{for} \quad d_i > 2R_i + \rho, \\ \frac{1}{\epsilon_W}(G_i - G_i^{'})(2R_i + \rho - d_i)^2\ , & \text{for} \quad 2R_i \leq d_i \leq 2R_i + \rho, \\ \frac{1}{\epsilon_W^{'}}(G_i - G_i^{'})(2R_i - d_i)\ , & \text{for} \quad d_i < 2R_i. \end{cases} .$$

where $\rho \approx h$, $\epsilon_W = \frac{\epsilon}{2}$ and $\epsilon_W^{'} = \frac{\epsilon^{'}}{2}$.



Imaginary body [2]

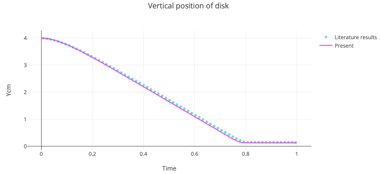
Total repulsion force applied on the body B_i :

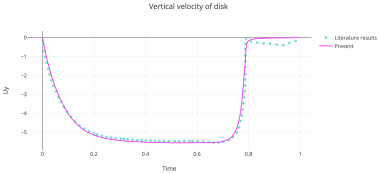
$$|\overrightarrow{F_i}| = \sum_{j=1, j\neq i}^{N} |\overrightarrow{F_{ij}}| + |\overrightarrow{F_i^W}|,$$

where N the number of bodies.

Simulation : Spherical body-bottom interaction

Simulation in 2D

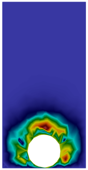




The differences in vertical velocity can be explained by a higher repulsion force intensity used for literature results [3].

Simulation in 3D
Plane $\{z = 0\}$

$t = 0.0$s

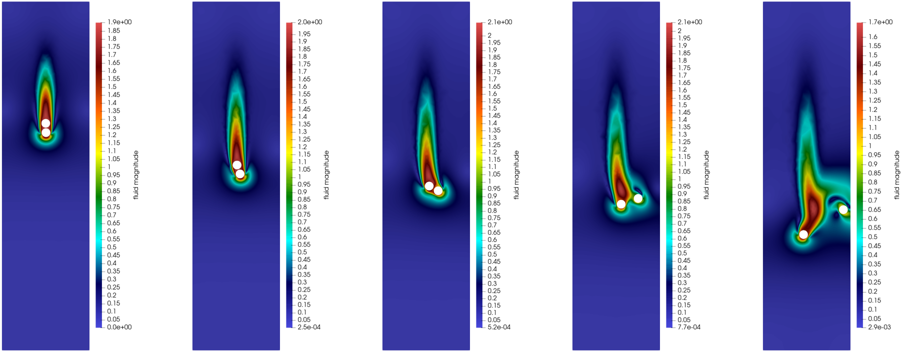


$t = 0.05$s

$t = 0.11$s

Simulation : Two disks interaction

Drafting, kissing and tumbling phenomenon in 2D

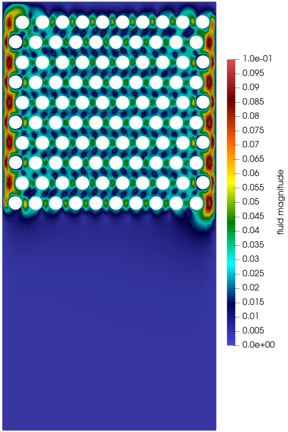


$t = 1.5$s $t = 2.0$s $t = 2.25$s $t = 2.5$s $t = 3.0$s

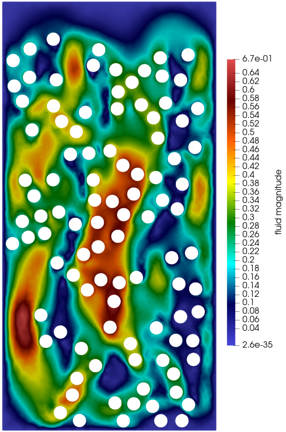
K. Usman, K. Walayat, R. Mahmood, et al. *Analysis of solid particles falling down and interacting in a channel with sedimentation using fictitious boundary method*, 2018.

Decheng Wan, Stefan Turek. *Direct Numerical Simulation of Particulate Flow via Multigrid FEM Techniques and the Fictitious Boundary Method*, 2004.

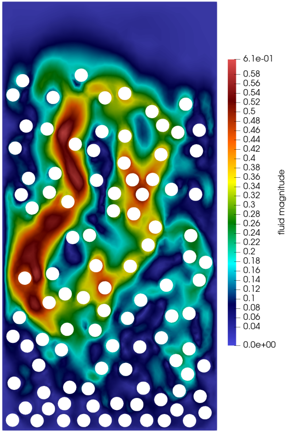
Simulation : 100 disks interaction



$t = 0.1\text{s}$



$t = 5.0\text{s}$

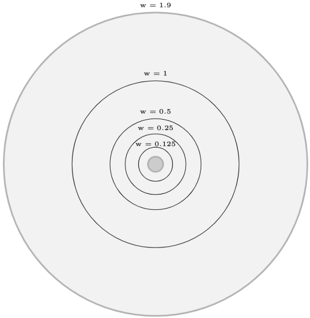


$t = 7.0\text{s}$

L.H.Juarez, R.Glowinski, T.W.Pan. *Numerical simulation of fluid flow with moving and free boundaries*, 2004.

Narrow band fast marching method :

- FMM computes the distance function $D_i(X) : \mathbb{R}^d \longrightarrow \mathbb{R}$ for each point of the mesh starting from a front.
- Narrow band approach : $D_i(X)$ only computed in a near neighborhood of bandwidth w around the front.



Bandwidth	Execution time in 2D	Execution time in 3D
1.9	1956.92 ms	22854.00 ms
1.0	479.00 ms	3930.71 ms
0.5	143.60 ms	686.39 ms
0.25	52.72 ms	184.69 ms
0.125	26.01 ms	93.18 ms

Performance of narrow band approach

J.A. Sethian. *A fast marching level set method for monotonically advancing fronts*,1996.

Collisions with complex shaped rigid bodies

Reformulation of $\overrightarrow{F_{ij}}$ for body-body and $\overrightarrow{F_i^W}$ for body-wall interaction :

$$|\overrightarrow{F_{ij}}| = \begin{cases} 0\,, & \text{for} \quad d_{ij} > \rho, \\ \frac{1}{\epsilon}(\arg\min_{X\in\partial B_i} D_j(X) - \arg\min_{X\in\partial B_j} D_i(X))(\rho - d_{ij})^2\,, & \text{for} \quad 0 \le d_{ij} \le \rho. \end{cases}$$

$$|\overrightarrow{F_i^W}| = \begin{cases} 0\,, & \text{for} \quad d_i > \rho, \\ \frac{1}{\epsilon_W}(\arg\min_{X\in\partial B_i} D_\Omega(X) - \arg\min_{X\in\partial\Omega} D_i(X))(\rho - d_i)^2\,, & \text{for} \quad 0 \le d_i \le \rho. \end{cases}$$

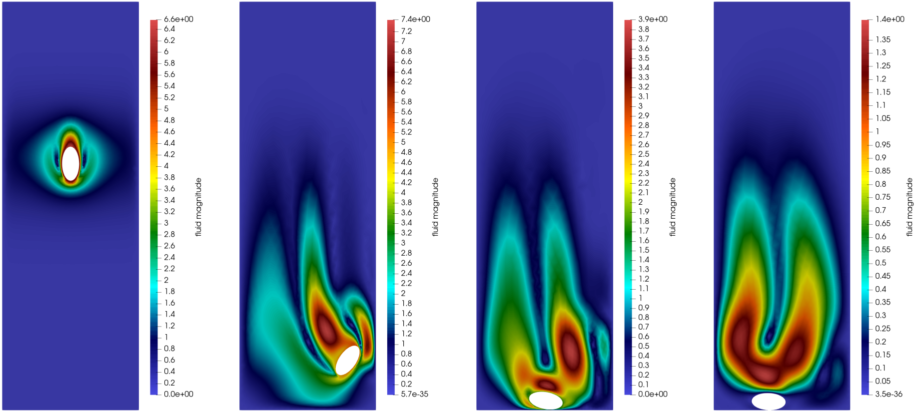
Angular momentum of $\overrightarrow{F_i} = \overrightarrow{F_{ij}} + \overrightarrow{F_i^W}$ for body-body and body-wall interaction :

$$\frac{d[RJ_iR^T\omega_i]}{dt} = T_i - \overrightarrow{Gx_r} \times \overrightarrow{F_i},$$

where G the mass center of the body and x_r the point where $\overrightarrow{F_i}$ applies on the body B_i.

Tsorng-Whay Pan, Roland Glowinski, Giovanni P.Galdi. *Direct simulation of the motion of a settling ellipsoid in Newtonian fluid*, 2001.

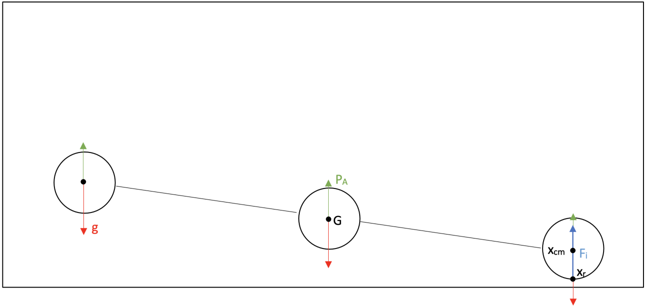
Simulation : Ellipse-bottom interaction



$t = 0.1$s $t = 0.5$s $t = 0.75$s $t = 1.25$s

L.H.Juarez, R.Glowinski, T.W.Pan. *Numerical simulation of fluid flow with moving and free boundaries*, 2004.

Collisions with articulated rigid bodies



Newton's equation describing the angular velocity :

$$\frac{d[RJ_iR^T\omega_i]}{dt} = T_i - \overrightarrow{Gx_r} \times \overrightarrow{F_i} + \overrightarrow{Gx_{cm}} \times \overrightarrow{(g - P_A)},$$

where G the mass center of the swimmer, x_r the point where $\overrightarrow{F_i}$ applies on the body, x_{cm} the mass center of one body, g the gravity vector and P_A the buoyancy.

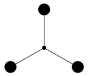
Other results and present work

Simulations and benchmarks

All algorithms have been validated by test cases from literature. The influence of the number of interacting bodies on the execution time has been analyzed. Other simulations have been performed : falling complex body, falling three-sphere swimmer, etc.

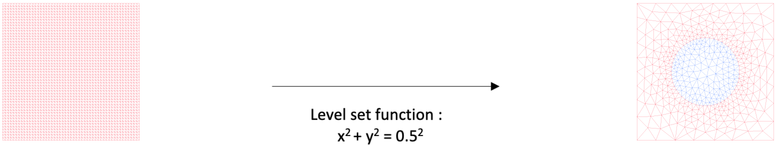
Other internship results

Numerical resolution of the Stokes equations and analysis of the motion of the three-sphere planar swimmer. Small numerical errors and displacements close to the theoretical ones.



Present work

Code parallelization and insertion of bodies into a mesh via level set functions using Mmg software for remeshing.



Initial mesh

Final mesh

Conclusion and perspectives

Near future work

- Simulations of three-sphere swimmer near the wall, simulations in three dimensions : interacting multi spheres, ellipsoid - wall interaction.
- Developing a method to determine the level set function of a body of arbitrary shape.
- Writing a paper on collision modeling, algorithms and simulations.

Farther future work

- Modeling and simulation of direct contact.
- Interactions between deformable bodies.

Conclusion

- Our collision algorithms can model and simulate contact between bodies of arbitrary shapes. The results are in good agreement with the literature.
- We worked on methods to increase the efficiency of the algorithms : narrow band fast marching method, insertion of bodies into a mesh via level set functions.

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